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一类交错网格的 Gauss 型格式

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摘要:本文在交错网格的情况下,利用 Gauss 型求积公式构造了一类不需解 Riemann 问题的求解一维单个双曲守恒律的二阶显式 Gauss 型差分格式,证明了该格式在 CFL 条件限制下为 TVD 格式,并证明了这类格式的收敛性,然后将格式推广到方程组的情形. 由于在交错网格的情况下构造的这类差分格式,不需要求解 Riemann 问题,因此这类格式与诸如 Harten 等的 TVD 格式相比具有如下优点:由于不需要完整的特征向量系,因此可用于求解弱双曲方程组,计算更快、编程更加简便等.

关键词:守恒;TVD;通量;差分格式;收敛性;交错网格
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本文研究在交错网格下利用 Gauss 型差分格式求解一维单个双曲守恒律初值问题:

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial f(u)}{\partial x} = 0, & t > 0, x \in \mathbf{R}; \\ u(x,0) = u_0(x), & x \in \mathbf{R}, \end{cases} \quad f \in C^2(\mathbf{R}). \tag{1}$$

本文在交错网格的情况下,利用 Gauss 型求积公式构造了一类求解双曲守恒律的时空一致二阶显式 Gauss 型差分格式,该格式在 CFL 条件限制下为收敛的 TVD 格式,然后将格式推广到方程组的情形,我们得到的数值试验结果是很令人满意的. 由于在交错网格的情况下构造的这类 Gauss 型差分格式,不需要求解 Riemann 问题,因此这类格式具有计算简单、工作量少、编程简便等特点.

记空间步长为 h,时间步长为 k,步长比 $\lambda = k/h$, $x_j = jh$, $t_n = nk$, $x_{j+1/2} = \left(j + \frac{1}{2}\right)h$, $I_j = [x_{j-1/2}, x_{j+1/2}]$, $I_{j+1/2} = [x_j, x_{j+1}]$. 下面描述交错网格的 Gauss 型格式的构造. 用 v_j^n 和 $v_{j+1/2}^n$ 分别表示 $u(x,t)$ 在时间层 t_n 时在格子 $I_j = [x_{j-1/2}, x_{j+1/2}]$ 和 $I_{j+1/2} = [x_j, x_{j+1}]$ 的平均值,即

$$v_j^n = \frac{1}{h}\int_{I_j} u(x,t_n)\,\mathrm{d}x, \quad v_{j+1/2}^n = \frac{1}{h}\int_{I_{j+1/2}} u(x,t_n)\,\mathrm{d}x. \tag{2}$$

在每个时间层上利用线性逼近对平均值函数进行重构:

$$v(x,t_n) = L_j(x,t_n) = v_j^n + \frac{x - x_j}{h} v_j', \quad x_{j-1/2} \leqslant x \leqslant x_{j+1/2}, \tag{3}$$

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其中 $\frac{1}{h}v_j' = \frac{\partial u(x, t_n)}{\partial x}\Big|_{x=x_j} + O(h)$. 利用重构函数代替方程(1)中的函数 $u(x,t)$, 并在区间 $[t_n, t_{n+1}] \times I_{j+1/2}$ 上积分, 得到

$$v_{j+1/2}^{n+1} = \frac{1}{2}(v_{j+1}^n + v_j^n) - \frac{1}{8}(v_{j+1}' - v_j') - \frac{1}{h}\int_{t_n}^{t_{n+1}}(f(v(x_{j+1},t)) - f(v(x_j,t)))\,dt. \quad (4)$$

利用两点 Gauss 求积公式逼近(4)中的积分, 我们得到在交错网格情况下的 Gauss 型格式:

$$\begin{cases} v_{j+1/2}^{n+1} = \frac{1}{2}(v_{j+1}^n + v_j^n) - \frac{1}{8}(v_{j+1}' - v_j') - \lambda(h_{j+1}^n - h_j^n), \\ h_j^n = \frac{1}{2}(f(v_j^n - h_1 f_j') + f(v_j^n - h_2 f_j')), \end{cases} \quad (5)$$

其中 $\frac{1}{h}f_j' = \frac{\partial f(v(x,t_n))}{\partial x}\Big|_{x=x_j} + O(h)$, $h_1 = \lambda\left(\frac{1}{2} - \frac{\sqrt{3}}{6}\right)$, $h_2 = \lambda\left(\frac{1}{2} + \frac{\sqrt{3}}{6}\right)$.

利用 Taylor 展开式, 容易证明:

定理 1 由(5)式定义的 Gauss 型格式除在函数 $u(x,t)$ 的驻点外是二阶格式.

定理 2 如果 Gauss 格式(5)中的数值导数 $v_j'/h, f_j'/h$ 满足以下条件

$$\begin{cases} v_j' = MM\left\{\alpha\Delta v_{j+1/2}^n, \frac{1}{2}(v_{j+1}^n - v_{j-1}^n), \alpha\Delta v_{j-1/2}^n\right\}, \\ f_j' = MM\left\{\alpha\Delta f_{j+1/2}^n, \frac{1}{2}(f_{j+1}^n - f_{j-1}^n), \alpha\Delta f_{j-1/2}^n\right\}, \end{cases} \quad (6)$$

那么, 当 $0 < \alpha < 4$ 在 CFL 条件(7)的限制下, 格式(5)为 TVD 格式.

$$\lambda\max_j |f'(v_j^n)| \le CFL \le \frac{1}{2\alpha}(\sqrt{4 + 4\alpha - \alpha^2} - 2). \quad (7)$$

证 由于 $v_{j+1/2}^{n+1} - v_{j-1/2}^{n+1} = \Delta v_{j+1/2}^n\left(\frac{1}{2} - \lambda\frac{\Delta g_{j+1/2}^n}{\Delta v_{j+1/2}^n}\right) + \Delta v_{j-1/2}^n\left(\frac{1}{2} + \lambda\frac{\Delta g_{j-1/2}^n}{\Delta v_{j-1/2}^n}\right)$, 其中

$$g_j^n = h_j^n + \frac{1}{8\lambda}v_j', \quad \Delta g_{j+1/2}^n = g_{j+1}^n - g_j^n. \quad (8)$$

要证(5)为 TVD 格式, 即 $TV(V^{n+1}) \equiv \sum_j |v_{j+1/2}^{n+1} - v_{j-1/2}^{n+1}| \le \sum |\Delta v_{j+1/2}^n| \equiv TV(V^n)$, 只要证 $\lambda\left|\frac{\Delta g_{j+1/2}^n}{\Delta v_{j+1/2}^n}\right| \le \frac{1}{2}$ 即可.

$$\lambda\left|\frac{\Delta g_{j+1/2}^n}{\Delta v_{j+1/2}^n}\right| \le \frac{\lambda}{2}\left(\left|\frac{f(v_{j+1}^n - h_1 f_{j+1}') - f(v_j^n - h_1 f_j')}{(v_{j+1}^n - h_1 f_{j+1}') - (v_j^n - h_1 f_j')}\right|\right.$$
$$\times\left|\frac{(v_{j+1}^n - h_1 f_{j+1}') - (v_j^n - h_1 f_j')}{\Delta v_{j+1/2}^n}\right| + \left|\frac{f(v_{j+1}^n - h_2 f_{j+1}') - f(v_j^n - h_2 f_j')}{(v_{j+1}^n - h_2 f_{j+1}') - (v_j^n - h_2 f_j')}\right|$$
$$\left.\times\left|\frac{(v_{j+1}^n - h_2 f_{j+1}') - (v_j^n - h_2 f_j')}{\Delta v_{j+1/2}^n}\right|\right) + \frac{1}{8}\left|\frac{\Delta v_{j+1/2}'}{\Delta v_{j+1/2}^n}\right|,$$

由(7)的 CFL 条件知: $\lambda\left|\frac{f(v_{j+1}^n - h_i f_{j+1}') - f(v_j^n - h_i f_j')}{(v_{j+1}^n - h_i f_{j+1}') - (v_j^n - h_i f_j')}\right| \le CFL, i = 1, 2$. 而 $\left|\frac{(v_{j+1}^n - h_i f_{j+1}') - (v_j^n - h_i f_j')}{\Delta v_{j+1/2}^n}\right| \le 1 + h_i\left|\frac{\Delta f_{j+1/2}'}{\Delta v_{j+1/2}^n}\right|, i = 1, 2$, 由(6)和(7)有

$$\left|\frac{\Delta v_{j+1/2}'}{\Delta v_{j+1/2}^n}\right| \le \max\left(\left|\frac{v_{j+1}'}{\Delta v_{j+1/2}^n}\right|, \left|\frac{v_j'}{\Delta v_{j+1/2}^n}\right|\right) \le \alpha,$$

$$\left|\frac{\Delta f_{j+1/2}'}{\Delta v_{j+1/2}^n}\right| \le \max\left(\left|\frac{f_{j+1}'}{\Delta v_{j+1/2}^n}\right|, \left|\frac{f_j'}{\Delta v_{j+1/2}^n}\right|\right) \le \frac{1}{\lambda}\alpha CFL.$$

所以 $\lambda\left|\frac{\Delta g_{j+1/2}^n}{\Delta v_{j+1/2}^n}\right| \le CFL\left(1 + \frac{1}{2}\alpha CFL\right) + \frac{1}{8}\alpha \le \frac{1}{2}$. 因此, Gauss 型格式(5)在(6)和(7)的限制

下为 TVD 格式.

定理 3　如果 Gauss 格式(5)中的数值导数 $v_j'/h, f_j'/h$ 满足以下条件

$$\begin{cases} 0 \le v_j' \,\mathrm{sgn}(\Delta v_{j+1/2}^n) \le \alpha \left|MM(\Delta\tilde v_{j+1/2}, \Delta\tilde v_{j-1/2})\right|, \quad |\alpha|\le 1,\\ \Delta \tilde v_{j+1/2} \equiv \Delta v_{j+1/2}^n \left[1-\lambda(1-4CFL^2)\left(\max\limits_x f''(v(x))\, \Delta v^n_{j+1/2}\right)^+\right]^+,\\ 0\le f_j' \,\mathrm{sgn}(\Delta v^n_{j+1/2}) \le \beta \left|MM(\Delta v^n_{j+1/2}, \Delta v^n_{j-1/2})\right|, \quad \beta \equiv \dfrac{\alpha\cdot CFL}{\lambda},\end{cases} \tag{9}$$

那么,在 CFL 条件(7)的限制下,Gauss 型格式(5)的解满足熵条件,即格式是收敛的.

证　用 $g(v)$ 表示由(8)定义的网格函数 $\{g_j^n\}$ 的分段线性插值函数:

$$g(v) = \frac{\Delta g^n_{j+1/2}}{\Delta v^n_{j+1/2}}(v-v_j^n)+g_j^n, \quad \min(v_j^n,v_{j+1}^n)\le v\le \max(v_j^n,v_{j+1}^n). \tag{10}$$

我们取凸熵函数 $U(u)=\frac{1}{2}u^2$ 及对应的熵通量 $F(u)=\int^u f'(u)\,U'(u)\,\mathrm{d}u$,令

$$v(s) = s v_j^n + (1-s) v_{j+1}^n, \quad v(0)=v_{j+1}^n, \quad v(1)=v_j^n;$$

$$v(r,s) = r v(s) + (1-r) v_{j+1}^n, \quad v(0,s)=v^n_{j+1}, \quad v(1,s)=v(s);$$

$$v_{j+1/2}(s) = \frac{1}{2}(v(s)+v^n_{j+1}) - \lambda(g^n_{j+1} - g(v(s))), \quad v_{j+1/2}(0)=v^n_{j+1}, \quad v_{j+1/2}(1)=v^{n+1}_{j+1/2};$$

$$v_{j+1/2}(r,s) = \frac{1}{2}(v(s)+v(r,s)) - \lambda(g(v(r,s)) - g(v(s))),$$

$$v_{j+1/2}(1,s)=v(s), \quad v_{j+1/2}(0,s)=v_{j+1/2}(s).$$

引入与熵通量 $F(u)$ 相容的数值熵通量 $G_j = F(v_j^n)+v_j^n(g(v_j^n)-f(v_j^n))$,并记

$$\begin{aligned} U(v^{n+1}_{j+1/2}) &= U\left(\frac{1}{2}(v^n_{j+1}+v^n_j) - \frac{1}{8}(v'_{j+1}-v'_j) - \lambda(h^n_{j+1}-h^n_j)\right)\\ &= \frac{1}{2}(U(v^n_{j+1})+U(v^n_j)) - \lambda(G_{j+1}-G_j) + R_{j+1/2}, \end{aligned}$$

其中 $R_{j+1/2} = U(v^{n+1}_{j+1/2}) - \frac{1}{2}(U(v^n_{j+1})+U(v^n_j)) + \lambda(G_{j+1}-G_j)$. 为了证明格式(5)满足熵条件,只需证明 $R_{j+1/2} \le 0$. 由于

$$\frac{\mathrm{d}v(s)}{\mathrm{d}s} = -\Delta v^n_{j+1/2}, \quad \frac{\mathrm{d}v_{j+1/2}(s)}{\mathrm{d}s} = -\left[\frac{1}{2} + \lambda g'(v(s))\right] \Delta v^n_{j+1/2}, \quad g'(v(s)) = g'(v(r,s)) = \frac{\Delta g^n_{j+1/2}}{\Delta v^n_{j+1/2}}$$

所以

$$\begin{aligned} \frac{\mathrm{d}U(v_{j+1/2}(s))}{\mathrm{d}s} &= -U'(v_{j+1/2}(s))\left[\frac{1}{2}+\lambda g'(v(s))\right]\Delta v^n_{j+1/2}\\ &= -v_{j+1/2}(s)\left[\frac{1}{2}+\lambda g'(v(s))\right]\Delta v^n_{j+1/2}; \end{aligned}$$

$$\frac{\mathrm{d}U(v(s))}{\mathrm{d}s} = -U'(v(s))\,\Delta v^n_{j+1/2} = -v(s)\,\Delta v^n_{j+1/2} = -v(s)\,\Delta v^n_{j+1/2}.$$

记 $G(v(s)) = F(v(s)) + v(s)(g(v(s)) - f(v(s)))$,并令

$$R(s) = U(v_{j+1/2}(s)) - \frac{1}{2}(U(v^n_{j+1})+U(v(s))) + \lambda(G_{j+1} - G(v(s))),$$

则 $R(0)=0, R(1)=R_{j+1/2}$,所以

$$\begin{aligned} R_{j+1/2} = \int_0^1 \frac{\mathrm{d}R(s)}{\mathrm{d}s}\mathrm{d}s = -\Delta v^n_{j+1/2} \int_0^1 \Big\{ & U'(v_{j+1/2}(s))\left[\frac{1}{2}+\lambda g'(v(s))\right] \\ & - \frac{1}{2} U'(v(s)) - \lambda(g(v(s)) - f(v(s)) + v(s)\,g'(v(s))) \Big\}\mathrm{d}s \end{aligned}$$

$$
\begin{aligned}
&= -\Delta v^n_{j+1/2}\int_0^1\left\{[v_{j+1/2}(s)-v(s)]\left[\frac{1}{2}+\lambda g'(v(s))\right]-\lambda(g(v(s))-f(v(s)))\right\}\mathrm{d}s\\
&= -(\Delta v^n_{j+1/2})^2\int_0^1\int_0^s\left[\frac{1}{2}-\lambda g'(v(r,s))\right]\left[\frac{1}{2}+\lambda g'(v(r,s))\right]\mathrm{d}r\,\mathrm{d}s\\
&\quad+\lambda\Delta v^n_{j+1/2}\int_0^1[g(v(s))-f(v(s))]\mathrm{d}s\\
&= -\frac{1}{2}(\Delta v^n_{j+1/2})^2\left[\frac{1}{4}-\left(\lambda\frac{\Delta g^n_{j+1/2}}{\Delta v^n_{j+1/2}}\right)^2\right]+\lambda\int_{v^n_j}^{v^n_{j+1}}(g(v)-f(v))\mathrm{d}v\\
&= -\frac{1}{2}(\Delta v^n_{j+1/2})^2\left[\frac{1}{4}-\left(\lambda\frac{\Delta g^n_{j+1/2}}{\Delta v^n_{j+1/2}}\right)^2\right]+\frac{\lambda}{2}(g^n_{j+1}+g^n_j)\Delta v^n_{j+1/2}-\lambda\int_{v^n_j}^{v^n_{j+1}}f(v)\mathrm{d}v.
\end{aligned}
$$

记 $\gamma=\lambda f'((v^n_{j+1}+v^n_j)/2)$，则由 Taylor 展开式及梯形求积公式的误差估计式有

$$
\begin{aligned}
\frac{\lambda}{2}(g^n_{j+1}+g^n_j)\Delta v^n_{j+1/2}-\lambda\int_{v^n_j}^{v^n_{j+1}}f(v)\mathrm{d}v &= \frac{1}{8}(1-4\gamma^2)\left(\frac{v'_{j+1}+v'_j}{2\Delta v^n_{j+1/2}}\right)(\Delta v^n_{j+1/2})^2\\
&\quad+\lambda\frac{1+\gamma^2}{12}f''(v(\eta))(\Delta v^n_{j+1/2})^3;
\end{aligned}
$$

$$
\begin{aligned}
\lambda\Delta g^n_{j+1/2} &= \lambda\Delta f^n_{j+1/2}+\frac{1}{8}(1-4\gamma^2)\left(\frac{\Delta v'_{j+1/2}}{\Delta v^n_{j+1/2}}\right)\Delta v^n_{j+1/2}\\
&= \gamma\Delta v^n_{j+1/2}+\frac{1}{8}(1-4\gamma^2)\left(\frac{\Delta v'_{j+1/2}}{\Delta v^n_{j+1/2}}\right)\Delta v^n_{j+1/2}+O(\Delta v^n_{j+1/2})^3;
\end{aligned}
$$

$$
\begin{aligned}
(\lambda\Delta g^n_{j+1/2})^2 &= \left[\gamma^2+\frac{\gamma}{4}(1-4\gamma^2)\left(\frac{\Delta v'_{j+1/2}}{\Delta v^n_{j+1/2}}\right)+\frac{1}{64}(1-4\gamma^2)^2\left(\frac{\Delta v'_{j+1/2}}{\Delta v^n_{j+1/2}}\right)^2\right](\Delta v^n_{j+1/2})^2\\
&\quad+O(\Delta v^n_{j+1/2})^4
\end{aligned}
$$

所以，由 CFL 条件限制 $\gamma\leqslant 1/2$，及条件(9)有

$$
\begin{aligned}
R_{j+1/2} &= \frac{1-4\gamma^2}{8}(\Delta v^n_{j+1/2})^2\left\{-1+\gamma\frac{\Delta v'_{j+1/2}}{\Delta v^n_{j+1/2}}+\frac{1}{16}(1-4\gamma^2)\left(\frac{\Delta v'_{j+1/2}}{\Delta v^n_{j+1/2}}\right)^2+\left(\frac{v'_{j+1}+v'_j}{2\Delta v^n_{j+1/2}}\right)\right\}\\
&\quad+\lambda\frac{1+\gamma^2}{12}f''(v(\eta))(\Delta v^n_{j+1/2})^3+O(\Delta v^n_{j+1/2})^4\\
&\leqslant \frac{1-4\gamma^2}{8}(\Delta v^n_{j+1/2})^2\left\{-1+\gamma\frac{\Delta v'_{j+1/2}}{\Delta v^n_{j+1/2}}+\frac{1}{16}(1-4\gamma^2)\left(\frac{\Delta v'_{j+1/2}}{\Delta v^n_{j+1/2}}\right)^2+\left(\frac{v'_{j+1}+v'_j}{2\Delta v^n_{j+1/2}}\right)\right\}\\
&\quad+\frac{\lambda}{8}(\Delta v^n_{j+1/2})^3\max_x(f''(v(x)))\\
&\leqslant \frac{1-4\gamma^2}{8}(\Delta v^n_{j+1/2})^2\left\{\left(\frac{v'_{j+1}+v'_j}{2\Delta v^n_{j+1/2}}\right)+\left|\frac{\Delta v'_{j+1/2}}{2\Delta v^n_{j+1/2}}\right|-1+\left(\gamma-\frac{1}{2}\right)\left|\frac{\Delta v'_{j+1/2}}{\Delta V^n_{j+1/2}}\right|\right.\\
&\quad\left.+\frac{1}{16}(1-4\gamma^2)\left(\frac{\Delta v'_{j+1/2}}{\Delta v^n_{j+1/2}}\right)^2\right\}+\frac{\lambda}{8}(\Delta v^n_{j+1/2})^3\max_x(f''(v(x)))\\
&\leqslant \frac{1-4\gamma^2}{8}(\Delta v^n_{j+1/2})^2\left[\max\left(\frac{v'_{j+1}}{2\Delta v^n_{j+1/2}},\frac{v'_j}{2\Delta v^n_{j+1/2}}\right)-1\right]+\frac{\lambda}{8}(\Delta v^n_{j+1/2})^3\max_x(f''(v(x)))\leqslant 0
\end{aligned}
$$

所以，Gauss 型格式(5)的解满足熵条件，因此格式是收敛的.

考虑方程组的情形. 将求解单个双曲守恒律的差分格式(5)推广到求解方程组

$$
\begin{cases}
\dfrac{\partial U}{\partial t}+\dfrac{\partial F(U)}{\partial x}=0, & t>0;\\
U(x,0)=U_0(x), & x\in\mathbf{R},
\end{cases}
\tag{11}
$$

其中 $U=U(x,t)=(u^1(x,t),\cdots,u^m(x,t))^{\mathrm{T}}$，$F(U)=(f^1(U),\cdots,f^m(U))^{\mathrm{T}}$. 记(11)在$(x_j,t_n)$

处的近似解为 $V_j^n=(v_j^1,\cdots,v_j^m)^T$,将(5) 改写为向量形式,得到求解(11) 的差分格式:

$$V_{j+1/2}^{n+1}=\frac{1}{2}(V_{j+1}^n+V_j^n)-\frac{1}{8}(V_{j+1}^{'}-V_j^{'})-\lambda(H_{j+1}^n-H_j^n),$$

其中 $H_j^n=\frac{1}{2}(F(V_j^n-h_1F_j^{'})+F(V_j^n-h_2F_j^{'}))$.

数值微分 $V_j^{'}, F_j^{'}$ 的每一个分量按(6) 式的定义选取.

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A Class of Gauss Scheme with Staggered Grids

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Abstract: In this paper, a class of the second order accurate explicit Gauss schemes with staggered grids for the computation of solutions of single hyperbolic conservation laws in one dimension are presented, these schemes are Riemann solver-free and total variation diminishing and convergence under the restriction of CFL, these schemes are extended to system of hyperbolic conservation laws. Because these schemes are constructed under staggered grids and Riemann solver-free, the advantages of these schemes compared to other TVD schemes such as Harten's are: no complete set of eigenvectors is needed and hence weakly hyperbolic system can be solved, faster and programming is much simpler.

Keywords: Conservation laws; TVD; Flux; Convergence; Staggered grids