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Letter to the Editor

Letter to Editor: Regarding the numerical results in "A novel finite-difference converged ENO scheme for steady-state simulations of Euler equations", by Tian Liang and Lin Fu, Journal of Computational Physics, 519 (2024), 113386

Yan Tan [a], Jun Zhu [a], Chi-Wang Shu [b], Jianxian Qiu [c,*]

[a] *College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu 210016, PR China*
[b] *Division of Applied Mathematics, Brown University, Providence, RI 02912, USA*
[c] *School of Mathematical Sciences and Fujian Provincial Key Laboratory of Mathematical Modeling and High-Performance Scientific Computing, Xiamen University, Xiamen, Fujian 361005, PR China*

We have recently noticed the paper by Tian Liang and Lin Fu, "A novel finite-difference converged ENO scheme for steady-state simulations of Euler equations, Journal of Computational Physics, 519 (2024), 113386", hereafter referred to as [1], which considers a new finite difference converged ENO scheme for computing the steady-state problem. We have read the paper and have disagreements about the comparisons with the WENO-ZQ scheme [2], which was cited as reference [29] and termed CWENOZ in [1] and with the MR-WENO scheme [3–5], which were cited as references [30, 36, 33] in [1]. Liang and Fu [1] report that the associated numerical results show that both WENO-ZQ and MR-WENO schemes can not converge to near machine zeros for steady state computation in many numerical examples, which is contrary to the results reported in our previous work. Therefore, we again compute these steady-state problems with the same parameters and settings as those in [1], and report the numerical results in this letter. In particular, the third-order strong-stability-preserving (SSP) Runge-Kutta method is applied for time discretization with the CFL number of 0.4, as specified in [1]. The parameter ε in the WENO weights is taken as 10^{-6} for both the WENO-ZQ scheme and the MR-WENO scheme, which is consistent with the setting in [1]. Besides, the globally Lax-Friedrichs flux splitting is taken for the computation. The linear weights are taken as $(d_0, d_1, d_2)^T = (0.98, 0.01, 0.01)^T$ and $\gamma_{l,l_2} = \frac{\bar{\gamma}_{l,l_2}}{\sum_{l_2=1}^{2} \bar{\gamma}_{l,l_2}}$, with $\bar{\gamma}_{1,2} = 1$, $\bar{\gamma}_{2,2} = 10$, $\bar{\gamma}_{1,3} = 1$, $\bar{\gamma}_{2,3} = 10$, $\bar{\gamma}_{3,3} = 100$ for the WENO-ZQ scheme and the MR-WENO scheme, respectively. All computations for each test problem are performed using the same program and only the mesh resolution is different. Our results show that all these problems can be computed well for the stated-steady problems, achieving near machine zero residual, by the WENO-ZQ scheme and the MR-WENO scheme, respectively.

The codes for WENO-ZQ and MR-WENO are attached as Research Data in Journal of Computational Physics. Please use ifort -r8 -o WENO_ZQ.f to compile the code. The corresponding numerical results are given below.

Example 4.3 in [1]. Oblique steady shock wave. In this example, Liang and Fu reported that "It can be seen that neither the average residual of TENO5 nor CWENOZ schemes could converge close to machine zero in this benchmark problem. The MRWENO scheme drives the residuals down to near machine zero at the coarse resolution of 240×120, except at the resolutions of 480×240 and 960×480". However, in Fig. 1, our numerical results show that both WENO-ZQ and MR-WENO can drive the residuals down to near machine zero.

Example 4.4 in [1]. Regular shock reflection problem. In this example, Liang and Fu reported that "It can be seen that although the average residual of CWENOZ can be reduced to 10^{-11} at the resolution of 160×40, when increasing the mesh resolution to 640×160 and 1280×320, the CWENOZ scheme presents uncertain convergence properties. The MRWENO scheme fails to converge

* Corresponding author.
E-mail addresses: tanyan@nuaa.edu.cn (Y. Tan), zhujun@nuaa.edu.cn (J. Zhu), chi-wang_shu@brown.edu (C.-W. Shu), jxqiu@xmu.edu.cn (J. Qiu).

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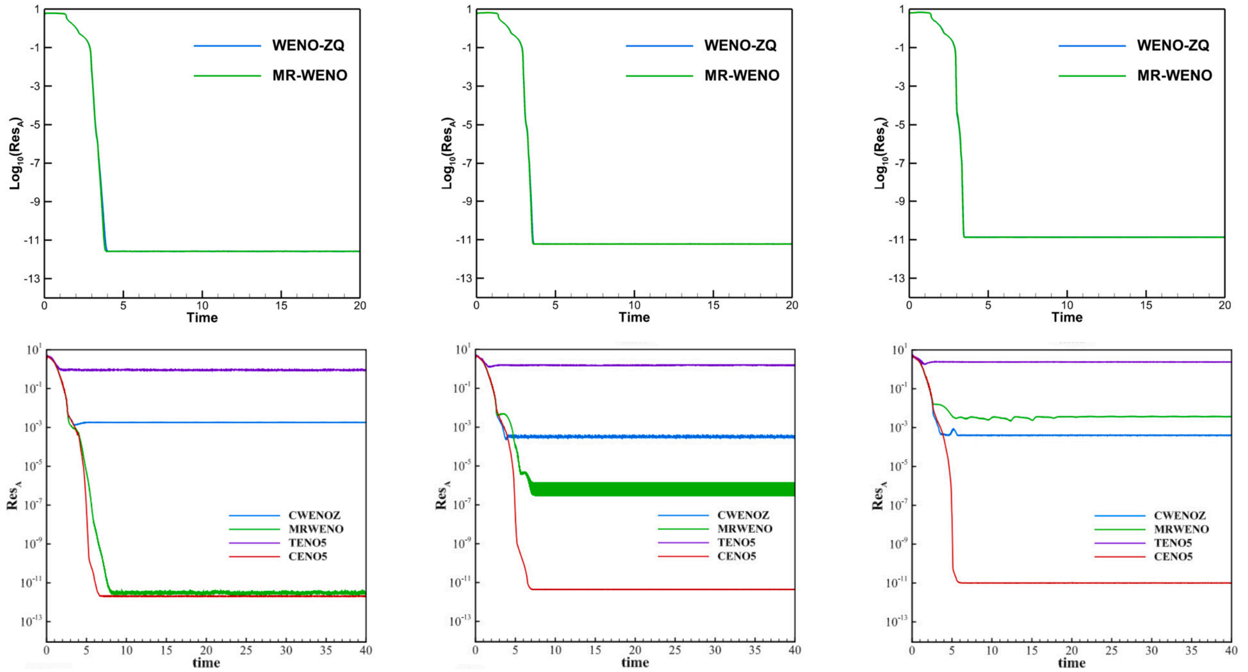

Fig. 1. The oblique steady shock wave problem. Top: the evolution of the average residue; bottom: results in [1]. From left to right: the mesh resolutions of 240×120, the mesh resolutions of 480×240, the mesh resolutions of 960×480.

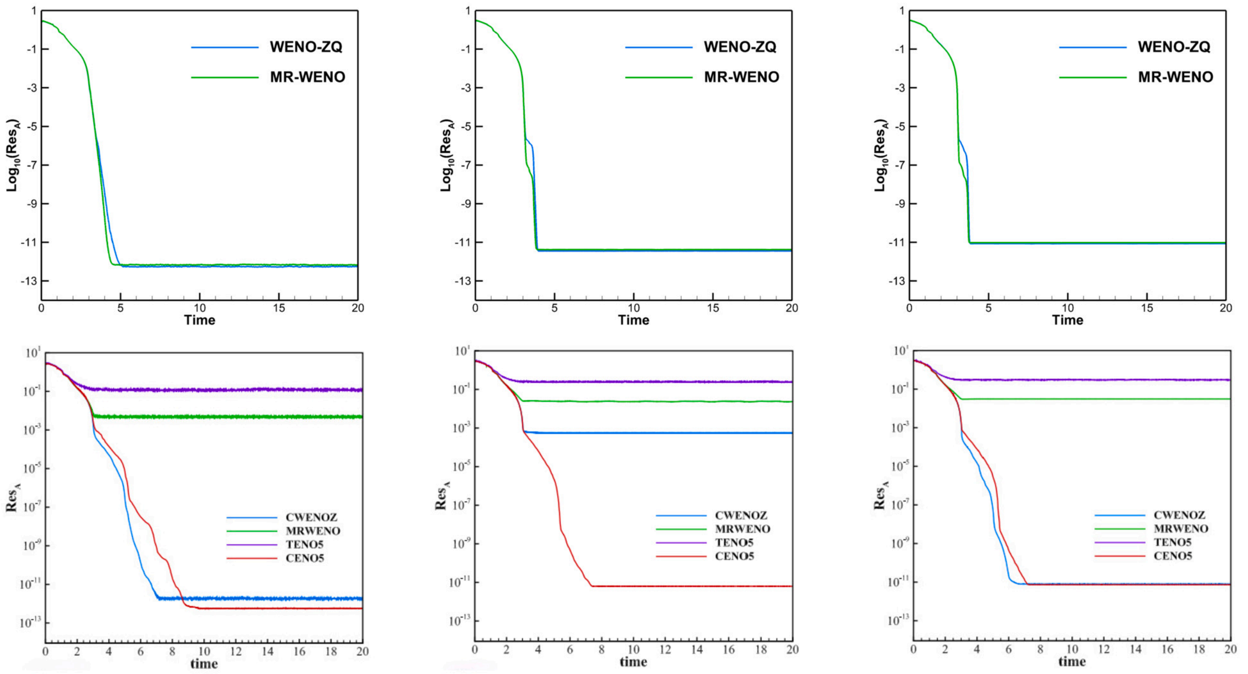

Fig. 2. The regular shock reflection problem. Top: the evolution of the average residue; bottom: results in [1]. From left to right: the mesh resolutions of 160×40, the mesh resolutions of 640×160, the mesh resolutions of 1280×320.

at all three tested resolutions". However, in Fig. 2, our numerical results show that both WENO-ZQ and MR-WENO can drive the residuals down to near machine zero.

Example 4.5 in [1]. Supersonic flow past a long plate. In this example, Liang and Fu reported that "For the MRWENO scheme, the average residual converges to 10^{-5} and 10^{-8} at the resolutions of 160×80 and 640×320, respectively. On the contrary, the average

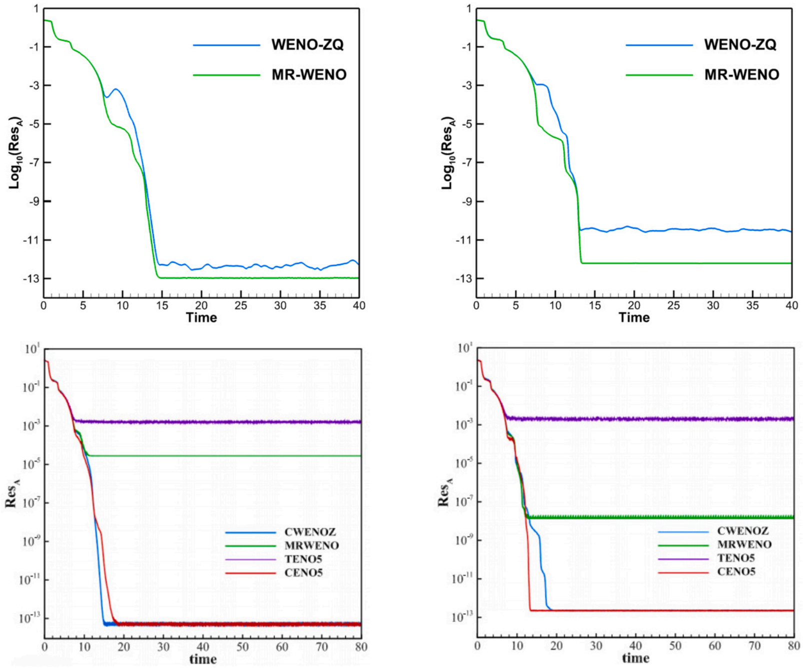

Fig. 3. The supersonic flow past a long plate. Top: the evolution of the average residue; bottom: results in [1]. From left to right: the mesh resolutions of 160×80, the mesh resolutions of 640×320.

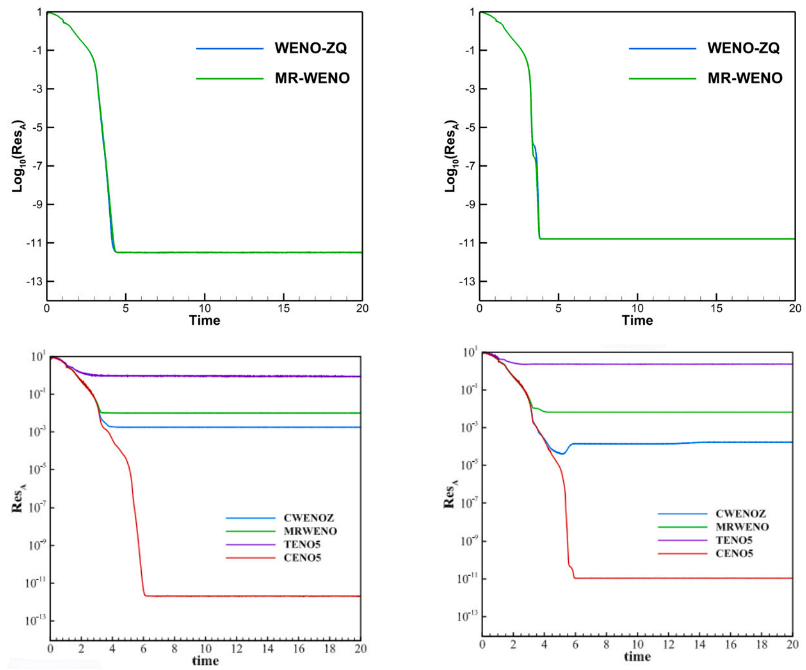

Fig. 4. The heterolateral shock interaction problem. Top: the evolution of the average residue; bottom: results in [1]. From left to right: the mesh resolutions of 240×120, the mesh resolutions of 960×480.

residual of the CWENOZ scheme and the present CENO5 scheme could settle down to machine zero". However, in Fig. 3, numerical results show that both WENO-ZQ and MR-WENO can drive the residuals down to near machine zero.

Example 4.6 in [1]. Heterolateral shock interaction problem. In this example, Liang and Fu reported that "It can be seen that the average residual of TENO5, MRWENO and CWENOZ schemes hangs at a high truncation error level. In comparison, the present CENO5 scheme can solve this problem successfully without slight post-shock oscillations in density distribution and the corresponding

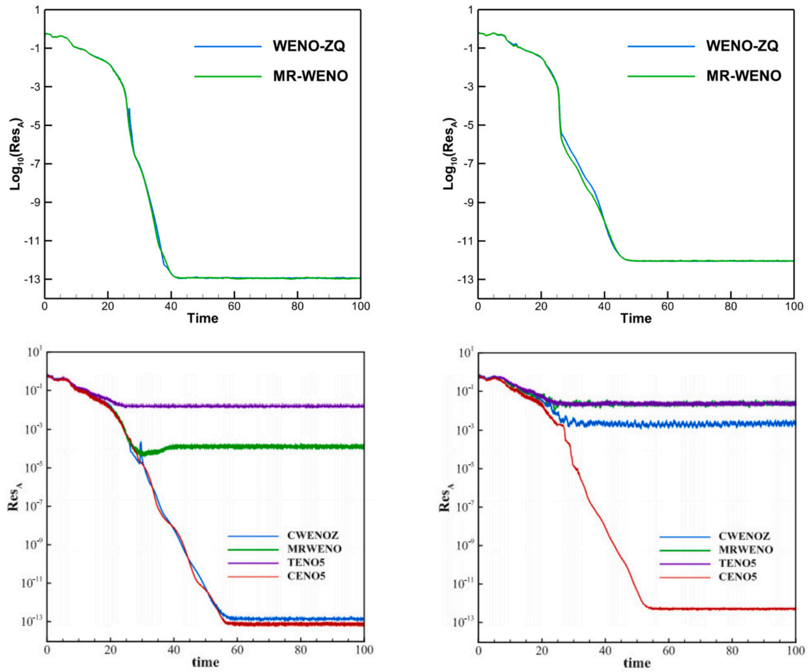

Fig. 5. The ipsilateral shock interaction problem. Top: the evolution of the average residue; bottom: results in [1]. From left to right: the mesh resolutions of 160×80, the mesh resolutions of 640×320.

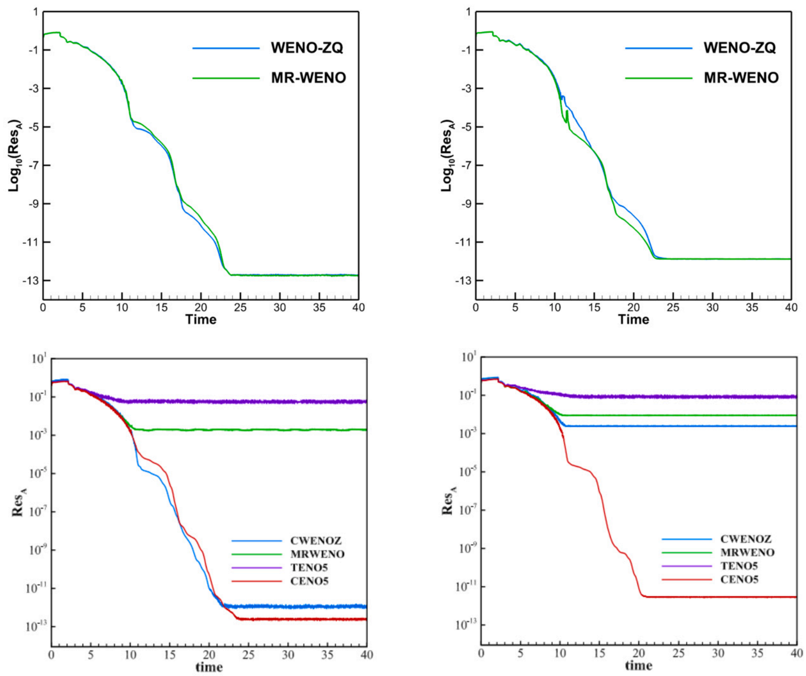

Fig. 6. The multiple shock reflection problem. Top: the evolution of the average residue; bottom: results in [1]. From left to right: the mesh resolutions of 320×40, the mesh resolutions of 1280×160.

convergence history of the residual could settle down to machine zero at both resolutions". However, in Fig. 4, our numerical results show that both WENO-ZQ and MR-WENO can drive the residuals down to near machine zero.

Example 4.7 in [1]. Ipsilateral shock interaction problem. In this example, Liang and Fu reported that "It can be observed that although the average residual of the CWENOZ scheme can settle down to a small value around 10^{-13} at the mesh resolution of

160×80, the average residual can only be reduced to around 10^{-3} at the mesh resolution of 640×320". However, in Fig. 5, our numerical results show that both WENO-ZQ and MR-WENO can drive the residuals down to near machine zero.

Example 4.8 in [1]. Multiple shock reflection problem. In this example, Liang and Fu reported that "It can be seen that the average residual from the present CENO5 scheme could settle down to machine zero at both resolutions, while the average residual of CWENOZ scheme can only be reduced to machine zero at the coarse resolution of 320×40". However, in Fig. 6, our numerical results show that both WENO-ZQ and MR-WENO can drive the residuals down to near machine zero.

CRediT authorship contribution statement

Yan Tan: Writing – original draft, Software. **Jun Zhu:** Writing – original draft, Software, Methodology, Conceptualization. **Chi-Wang Shu:** Writing – review & editing, Methodology, Conceptualization. **Jianxian Qiu:** Writing – review & editing, Supervision, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

We have uploaded the codes for WENO-ZQ and MR-WENO methods.

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